Determining the Thickness and Refractive Index of a Mirror

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When a laser beam reflects from a back surface glass mirror and falls on a screen, a pattern of discrete bright spots is created by partial reflection and refraction of the light at the air-glass interface and reflection at the mirror surface (Fig. 1). This paper explains how this phenomenon can be used to determine the refractive index and the thickness of the glass with a simple measurement. It is possible to utilize this experiment for geometrical optics labs and moreover it would be a nice practice for Physics Olympians.

Figure 2 shows the reflection and refraction of a laser beam from a horizontal mirror before hitting a vertical screen. Beam number 2 is the main one, so for most of the incident angles it is expected to be the brightest one. Beam number 1 is reflected from the top surface of the glass without entering in it; therefore, at high-incident angles it becomes brighter than beam number 1. In fact, it is possible to use Fresnel equations to make an estimate about the intensities of reflected and transmitted rays depending on the incident angle. Other rays’ intensities decrease as they make more reflections and refractions (Fig. 1).

The distance $x$ between the bright spots on the screen depends on the incoming angle $\theta$, thickness $t$, and the refractive index $n$ of the glass. Therefore, by measuring $x$ for different $\theta$ values, one can determine $n$ and $t$.

From Fig. 2 the relation between $x$ and $t$ can be determined as

$$2t \tan \alpha = x \tan \theta.$$  \hspace{1cm} (1)

By using Snell’s law,

$$\sin \theta = n \sin \alpha,$$  \hspace{1cm} (2)

we get the relation between $x$, $t$, and $\theta$:

$$x = \frac{2t \cos \theta}{\sqrt{n^2 - \sin^2 \theta}}.$$  \hspace{1cm} (3)

It is possible to rearrange this equation to obtain a relation that leads to a linear graph:

$$\frac{1}{x^2} = \frac{n^2 - 1}{4t^2} \frac{1}{\cos^2 \theta} + \frac{1}{4t^2}.$$  \hspace{1cm} (4)

If we plot $\frac{1}{x^2}$ versus $\frac{1}{\cos^2 \theta}$ graph, the $y$-intercept $\frac{1}{4t^2}$ will only depend on the thickness. After determining $t$, the slope $\frac{n^2 - 1}{4t^2}$ of the graph can be used to find the refractive index. If we send the laser beam at 30° angle of incidence to a regular glass mirror, which has a refractive index around 1.5 and a thickness around 0.25 cm, the distance $x$ between the spots on the screen is around 0.31 cm. Therefore, a graph paper posted on the screen can be used to measure $x$ easily.

This is a nice experiment that does not require advanced mathematical knowledge. However, my experience shows that even learning how to use Eq. (3) to determine both $t$ and $n$ simultaneously can be challenging for a regular college freshman. Therefore, I prefer to give a full explanation if I only want my students to improve their measurement skills and witness a nice demonstration. On the other hand, for advanced students I only give the setup, even without showing the pattern on the screen, and ask them to find the thickness and the refractive index of the glass. It is possible to choose a method of presenting this experiment between these two extremes to fit the level of the student.
Acknowledgments

I learned this experiment from my high school teacher Ayhan Uslu and my physics tutor Sinan Arslan at Samanyolu High School of Science, Ankara, Turkey. I would like to thank Dr. Pulak Dutta and Dr. Arthur Schmidt for their comments on the manuscript.

References


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